Combinational Circuit Design: Practice

Outline

- 1. Derivation of efficient HDL description
- 2. Operator sharing
- 3. Functionality sharing
- 4. Layout-related circuits
- 5. General circuits

1. Derivation of efficient HDL description

- Think "H", not "L", of HDL
- Right way:
 - Research to find an efficient design ("domain knowledge")
 - Develop VHDL code that accurately describes the design
- Wrong way:
 - Write a C program and covert it to HDL

Sharing

- Circuit complexity of VHDL operators varies
- Arith operators
 - Large implementation
 - Limited optimization by synthesis software
- "Optimization" can be achieved by "sharing" in RT level coding
 - Operator sharing
 - Functionality sharing

An example 0.55 um standard-cell CMOS implementation

width	VHDL operator									
	nand	xor	$>_a$	$>_d$	=	+1 $_a$	$+1_d$	$+_a$	$+_d$	mux
	area (gate count)									
8	8	22	25	68	26	27	33	51	118	21
16	16	44	52	102	51	55	73	101	265	42
32	32	85	105	211	102	113	153	203	437	85
64	64	171	212	398	204	227	313	405	755	171
delay (ns)										
8	0.1	0.4	4.0	1.9	1.0	2.4	1.5	4.2	3.2	0.3
16	0.1	0.4	8.6	3.7	1.7	5.5	3.3	8.2	5.5	0.3
32	0.1	0.4	17.6	6.7	1.8	11.6	7.5	16.2	11.1	0.3
64	0.1	0.4	35.7	14.3	2.2	24.0	15.7	32.2	22.9	0.3
64	0.1	0.4	35.7	14.3	2.2	24.0	15.7	32.2	22.9	

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Chapter 7

2. Operator sharing

- "value expressions" in priority network and multiplexing network are mutually exclusively:
- Only one result is routed to output
- Conditional sig assignment (if statement)

```
sig_name <= value_expr_1 when boolean_expr_1 else
value_expr_2 when boolean_expr_2 else
value_expr_3 when boolean_expr_3 else
...
value_expr_n;
```

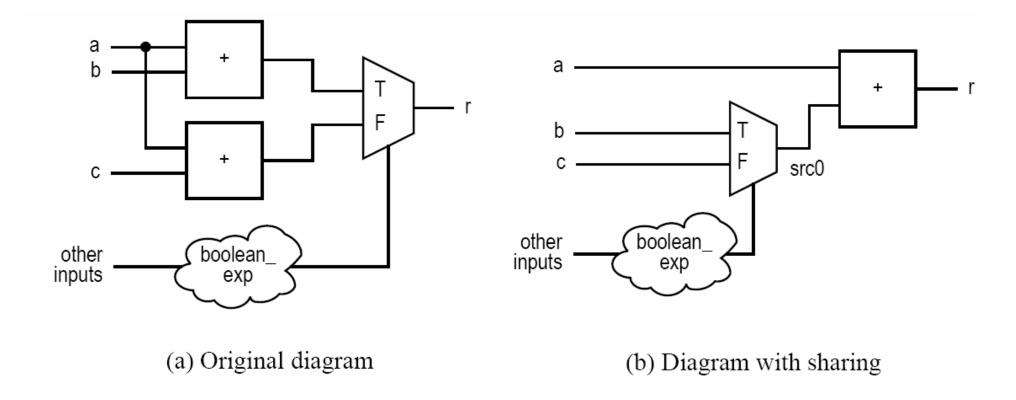
— Selected sig assignment (case statement)
with select_expression select
sig_name <= value_expr_1 when choice_1,</p>
value_expr_2 when choice_2,
value_expr_3 when choice_3,
....
value_expr_n when choice_n;

Example 1

Original code:

```
r <= a+b when boolean_exp else
a+c;</pre>
```

Revised code:



Area: 2 adders, 1 mux Delay:

$$\max(T_{adder}, T_{boolean}) + T_{mux}$$

Area: 1 adder, 1 mux Delay:

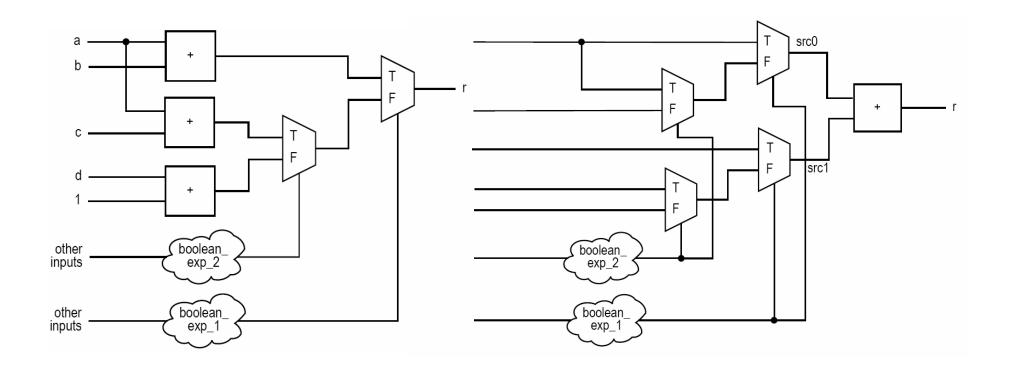
$$T_{boolean} + T_{mux} + T_{adder}$$

Example 2

 Original code: process(a,b,c,d,...) begin if boolean_exp_1 then r <= a+b;elsif boolean_exp_2 then $r \le a+c$; else r <= d+1;end if end process;

Revised code:

```
process(a,b,c,d,...)
begin
  if boolean_exp_1 then
   src0 \le a;
   src1 <= b;
 elsif boolean_exp_2 then
   src0 \le a;
   src1 <= c;
 else
   src0 \le d;
   src1 <= "0000001";
 end if;
end process;
r \le src0 + src1;
```

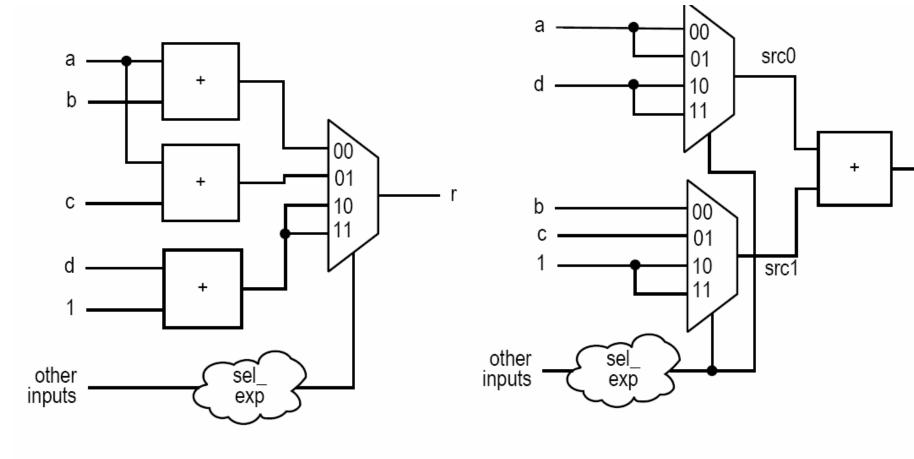


Area: 2 adders, 1 inc, 2 mux

Area: 1 adder, 4 mux

Example 3

 Original code: with sel select r <= a+b **when** "00", a+c when "01", d+1 when others; Revised code: with sel_exp select src0 <= a **when** "00"|"01", d when others; with sel_exp select src1 <= b **when** "00", c when "01", "0000001" when others; $r \le src0 + src1$;



(a) Original diagram

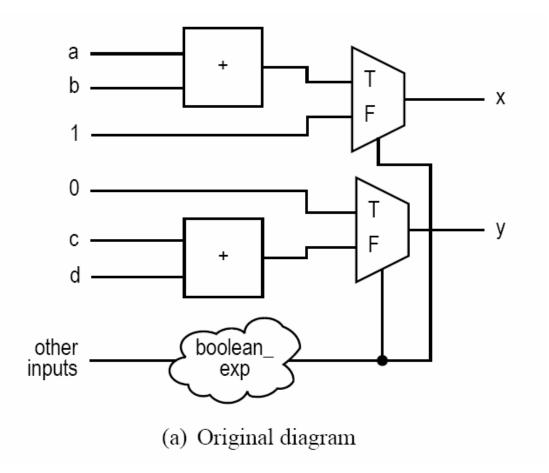
(b) Diagram with sharing

Area: 2 adders, 1 inc, 1 mux

Area: 1 adder, 2 mux

Example 4

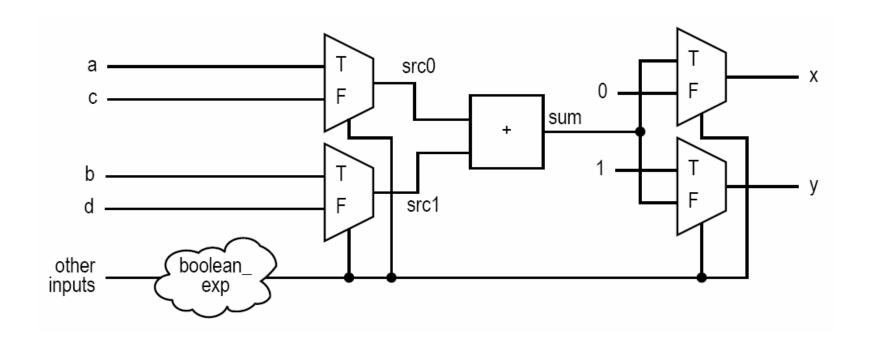
 Original code: process(a,b,c,d,...) begin if boolean_exp then $x \le a + b;$ y <= (others=>'0'); else $x \ll (others=>'1');$ y <= c + d;end if; end process;



Area: 2 adders, 2 mux

Revised code:

```
begin
  if boolean_exp then
   src0 \le a;
   src1 <= b;
   x \le sum;
   y <= (others=>'0');
 else
   src0 \ll c;
   src1 <= d;
   x <= (others=>'1');
   y <= sum;
 end if;
end process;
sum <= src0 + src1;
```



- Area: 1 adder, 4 mux
- Is the sharing worthwhile?
 - 1 adder vs 2 mux
 - It depends . . .

Summary

- Sharing is done by additional routing circuit
- Merit of sharing depends on the complexity of the operator and the routing circuit
- Ideally, synthesis software should do this

3. Functionality sharing

- A large circuit involves lots of functions
- Several functions may be related and have common characteristics
- Several functions can share the same circuit.
- Done in an "ad hoc" basis, based on the understanding and insight of the designer (i.e., "domain knowledge")
- Difficult for software it since it does not know the "meaning" of functions

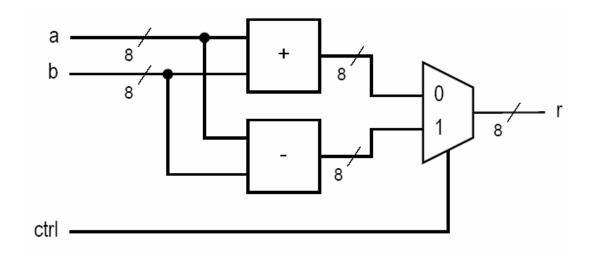
e.g., add-sub circuit

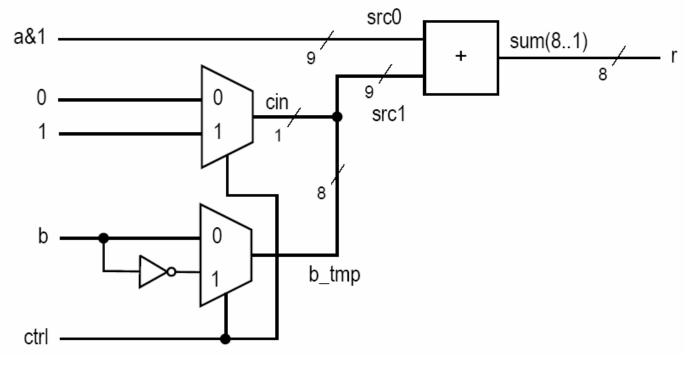
```
operation
                                              ctrl
library ieee;
use ieee.std_logic_1164.all;
                                                      a + b
                                               0
use ieee.numeric_std.all;
entity addsub is
                                                       a - b
   port (
      a,b: in std_logic_vector(7 downto 0);
      ctrl: in std_logic;
      r: out std_logic_vector(7 downto 0)
      );
end addsub;
architecture direct_arch of addsub is
   signal src0, src1, sum: signed(7 downto 0);
begin
   src0 <= signed(a);</pre>
   src1 <= signed(b);</pre>
   sum <= src0 + src1 when ctrl='0' else
           src0 - src1;
   r <= std_logic_vector(sum);
                                                           21
end direct_arch;
```

Observation: a – b can be done by a + b' + 1

- Manual injection of carry-in:
- Append an additional bit in right (LSB):

```
x_7x_6x_5x_4x_3x_2x_1x_01 and y_7y_6y_5y_4y_3y_2y_1y_0c_{in}
architecture manual_carry_arch of addsub is
   signal src0, src1, sum: signed(8 downto 0);
   signal b_tmp: std_logic_vector(7 downto 0);
   signal cin: std_logic; -- carry-in bit
begin
   src0 <= signed(a & '1');</pre>
   b_tmp <= b when ctrl='0' else
             not b;
   cin <= '0' when ctrl='0' else
           11:
   src1 <= signed(b_tmp & cin);</pre>
   sum \le src0 + src1;
   r <= std_logic_vector(sum(8 downto 1));
end manual_carry_arch;
```





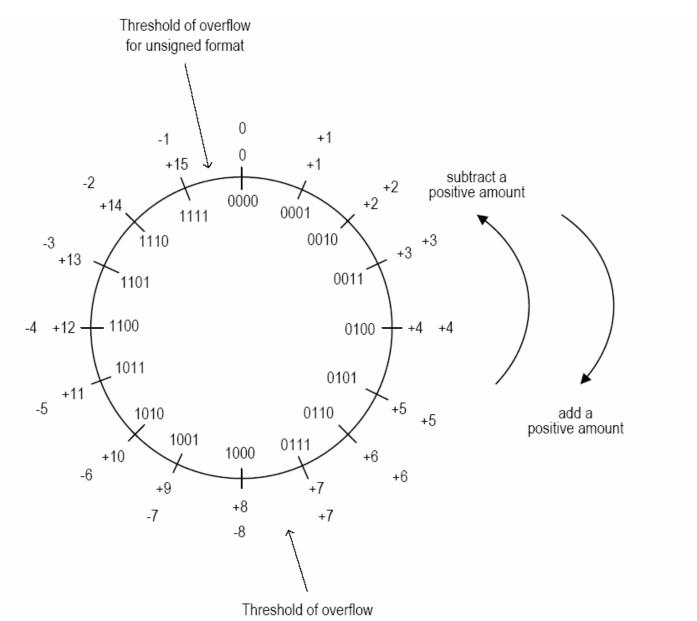
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Chapter 7

e.g., sign-unsigned comparator

```
library ieee;
   use ieee.std_logic_1164.all;
   use ieee.numeric_std.all;
   entity comp2mode is
      port (
         a,b: in std_logic_vector(7 downto 0);
         mode: in std_logic;
         agtb: out std_logic
      );
   end comp2mode;
   architecture direct_arch of comp2mode is
      signal agtb_signed, agtb_unsigned: std_logic;
   begin
   agtb_signed <= '1' when signed(a) > signed(b) else
                   ,o,:
   agtb_unsigned <= '1' when unsigned(a) > unsigned(b) els
                     00:
   agtb <= agtb_unsigned when (mode='0') else
           agtb_signed;
end direct_arch ;
```

Binary wheel



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for signed format

Observation:

- Unsigned: normal comparator
- Signed:
 - Different sign bit: positive number is larger
 - Same sign: compare remaining 3 LSBs This works for negative number, too!

```
E.g., 1111 (-1), 1100 (-4), 1001(-7)
111 > 100 > 001
```

- The comparison of 3 LSBs can be shared

e.g., Full comparator

```
library ieee;
use ieee.std_logic_1164.all;
entity comp3 is
   port (
       a,b: in std_logic_vector(15 downto 0);
       agtb, altb, aeqb: out std_logic
   );
end comp3;
architecture direct_arch of comp3 is
begin
   agtb <= '1' when a > b else
           ,0,:
   altb <= '1' when a < b else
           '0':
   aeqb \ll '1' when a = b else
           00:
end direct_arch;
```

```
architecture share1_arch of comp3 is
   signal gt, lt: std_logic;
begin
   gt <= '1' when a > b else
         ,0,;
   lt \le '1' when a < b else
         'O';
   agtb <= gt;
   altb <= lt;
   aeqb <= not (gt or lt);
end share1_arch;
```

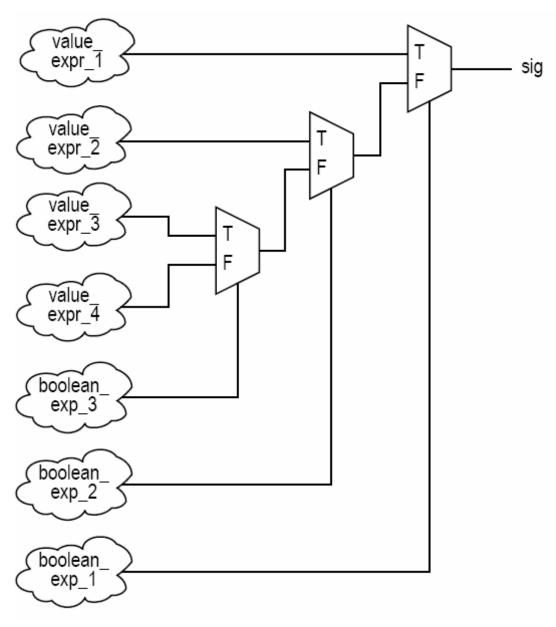
```
architecture share2_arch of comp3 is
   signal eq, lt: std_logic;
begin
   eq \leq '1' when a = b else
         '0';
   lt \le '1' when a < b else
          '0':
   aeqb <= eq;
   altb <= lt;
   agtb \le not (eq or lt);
end share2_arch;
```

Read 7.3.3 and 7.3.5

4. Layout-related circuits

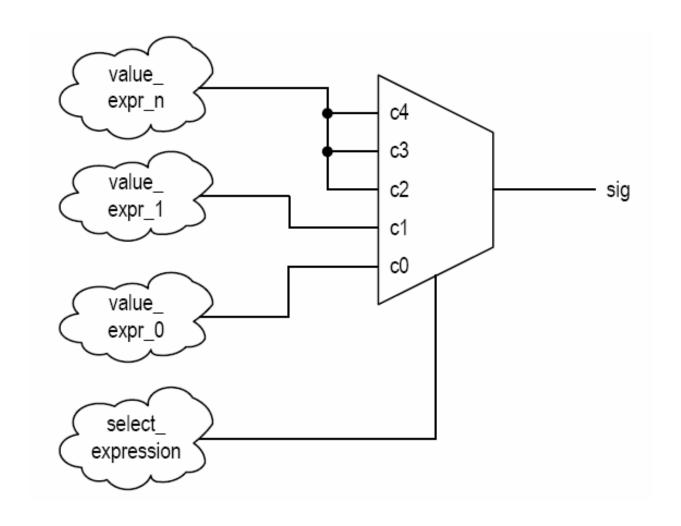
- After synthesis, placement and routing will derive the actual physical layout of a digital circuit on a silicon chip.
- VHDL cannot specify the exact layout
- VHDL can outline the general "shape"

- Silicon chip is a "square"
- "Two-dimensional" shape (tree or rectangular) is better than one-dimensional shape (cascadingchain)
- Conditional signal assignment/if statement form a single "horizontal" cascading chain
- Selected signal assignment/case statement form a large "vertical" mux
- Neither is ideal



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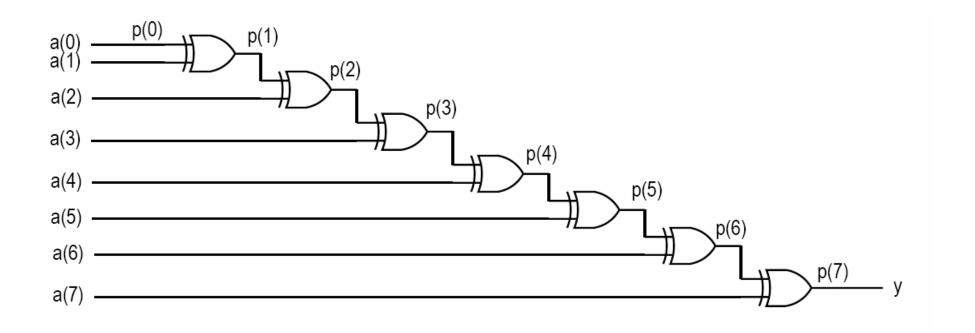
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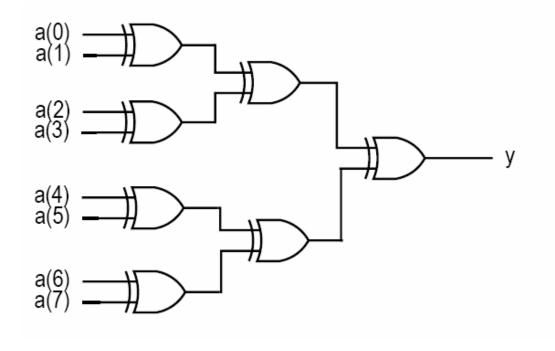


e.g., Reduced-xor circuit

```
a_7 \oplus a_6 \oplus a_5 \oplus a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0
library ieee;
use ieee.std_logic_1164.all;
entity reduced_xor is
   port (
      a: in std_logic_vector(7 downto 0);
      y: out std_logic
   );
end reduced_xor;
architecture cascade1_arch of reduced_xor is
begin
   y \le a(0) xor a(1) xor a(2) xor a(3) xor
         a(4) xor a(5) xor a(6) xor a(7);
end cascade1_arch;
```

```
architecture cascade2_arch of reduced_xor is
   signal p: std_logic_vector(7 downto 0);
begin
                                  p(0) \le 0, xor a(0);
   p(0) \le a(0);
   p(1) \le p(0) xor a(1);
   p(2) \le p(1) xor a(2);
   p(3) \le p(2) xor a(3);
   p(4) \le p(3) xor a(4);
   p(5) \le p(4) xor a(5);
   p(6) \le p(5) xor a(6);
   p(7) \le p(6) xor a(7);
   y \le p(7);
end cascade2_arch;
architecture cascade_compact_arch of reduced_xor is
   constant WIDTH: integer := 8;
   signal p: std_logic_vector(WIDTH-1 downto 0);
begin
   p \le (p(WIDTH-2 downto 0) & '0') xor a;
   y \le p(WIDTH-1);
end cascade_compact_arch;
```





- Comparison of n-input reduced xor
 - Cascading chain :
 - Area: (n-1) xor gates
 - Delay: (n-1)
 - Coding: easy to modify (scale)
 - -Tree:
 - Area: (n-1) xor gates
 - Delay: log₂n
 - Coding: not so easy to modify
 - Software should able to do the conversion automatically

e.g., Reduced-xor-vector circuit

```
y_0 = a_0

y_1 = a_1 \oplus a_0

y_2 = a_2 \oplus a_1 \oplus a_0

y_3 = a_3 \oplus a_2 \oplus a_1 \oplus a_0

y_4 = a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0

y_5 = a_5 \oplus a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0

y_6 = a_6 \oplus a_5 \oplus a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0

y_7 = a_7 \oplus a_6 \oplus a_5 \oplus a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0
```

Direct implementation

```
entity reduced_xor_vector is
   port (
      a: in std_logic_vector(7 downto 0);
      y: out std_logic_vector(7 downto 0)
   );
end reduced_xor_vector;
architecture direct_arch of reduced_xor_vector is
   signal p: std_logic_vector(7 downto 0);
begin
   y(0) \le a(0);
  y(1) \le a(1) xor a(0);
   y(2) \le a(2) xor a(1) xor a(0);
  y(3) \le a(3) xor a(2) xor a(1) xor a(0);
  y(4) \le a(4) xor a(3) xor a(2) xor a(1) xor a(0);
  y(5) \le a(5) xor a(4) xor a(3) xor a(2) xor a(1) xor a(0);
   y(6) \le a(6) xor a(5) xor a(4) xor a(3) xor a(2) xor a(1)
           xor a(0);
   y(7) \le a(7) xor a(6) xor a(5) xor a(4) xor a(3) xor a(2)
           xor a(1) xor a(0);
end direct_arch;
```

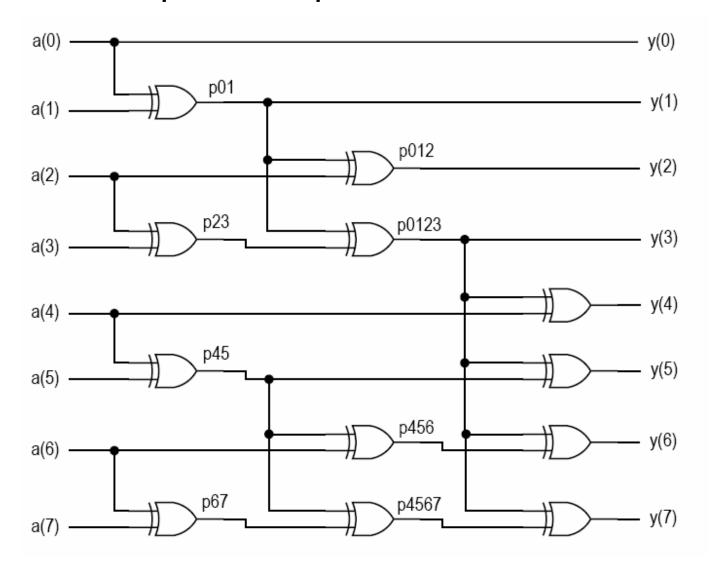
Functionality Sharing

```
architecture shared1_arch of reduced_xor_vector is
    signal p: std_logic_vector(7 downto 0);
 begin
   p(0) \le a(0);
p(1) \le p(0) xor a(1);
   p(2) \le p(1) xor a(2);
    p(3) \le p(2) xor a(3);
    p(4) \le p(3) xor a(4);
   p(5) \le p(4) xor a(5);
p(6) \le p(5) xor a(6);
   p(7) \le p(6) \text{ xor } a(7);
    y \le p;
 end shared1 arch:
 architecture shared_compact_arch of reduced_xor_vector is
    constant WIDTH: integer := 8;
    signal p: std_logic_vector(WIDTH-1 downto 0);
 begin
    p \le (p(WIDTH-2 downto 0) & '0') xor a;
    y \le p;
 end shared_compact_arch;
```

Direct tree implementation

```
architecture direct_tree_arch of reduced_xor_vector is
   signal p: std_logic_vector(7 downto 0);
begin
   v(0) \le a(0);
  y(1) \le a(1) xor a(0);
  y(2) \le a(2) xor a(1) xor a(0);
   y(3) \le (a(3) xor a(2)) xor (a(1) xor a(0));
  y(4) \le (a(4) xor a(3)) xor (a(2) xor a(1)) xor a(0);
  y(5) \le (a(5) xor a(4)) xor (a(3) xor a(2)) xor
           (a(1) xor a(0)):
   y(6) \le ((a(6) xor a(5)) xor (a(4) xor a(3))) xor
          ((a(2) xor a(1)) xor a(0));
   y(7) \le ((a(7) xor a(6)) xor (a(5) xor a(4))) xor
           ((a(3) xor a(2)) xor (a(1) xor a(0)));
end direct_tree_arch;
```

"Parallel-prefix" implementation



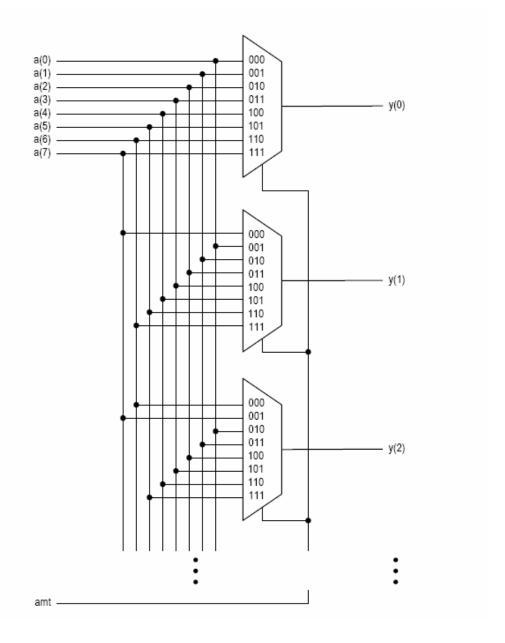
```
architecture optimal_tree_arch of reduced_xor_vector is
   signal p01, p23, p45, p67, p012,
           p0123, p456, p4567: std_logic;
begin
   p01 \le a(0) xor a(1);
   p23 \le a(2) xor a(3);
   p45 \le a(4) xor a(5);
   p67 \le a(6) xor a(7);
   p012 \le p01 xor a(2);
   p0123 \le p01 \text{ xor } p23;
   p456 \le p45 xor a(6);
   p4567 \le p45 \text{ xor } p67;
   y(0) \le a(0);
   y(1) \le p01;
   y(2) \le p012;
   y(3) \le p0123;
   y(4) \le p0123 xor a(4);
   y(5) \le p0123 xor p45;
   y(6) \le p0123 xor p456;
   y(7) \le p0123 \text{ xor } p4567;
end optimal_tree_arch;
```

- Comparison of n-input reduced-xor-vector
 - Cascading chain
 - Area: (n-1) xor gates
 - Delay: (n-1)
 - Coding: easy to modify (scale)
 - Multiple trees
 - Area: $O(n^2)$ xor gates
 - Delay: log₂n
 - Coding: not so easy to modify
 - Parallel-prefix
 - Area: O(nlog₂n) xor gates
 - Delay: log₂n
 - Coding: difficult to modify
 - Software is not able to convert cascading chain to parallel-prefix

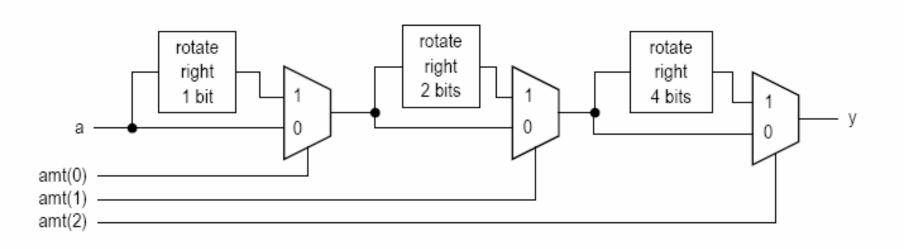
e.g., Shifter (rotating right)

Direct implementation

```
entity rotate_right is
   port (
      a: in std_logic_vector(7 downto 0);
      amt: in std_logic_vector(2 downto 0);
      y: out std_logic_vector(7 downto 0)
   );
end rotate_right;
architecture direct_arch of rotate_right is
begin
   with amt select
                                            when "000",
       y \le a
            a(0) & a(7 downto 1)
                                            when "001",
            a(1 downto 0) & a(7 downto 2) when "010",
            a(2 downto 0) & a(7 downto 3) when "011",
            a(3 downto 0) & a(7 downto 4) when "100",
            a (4 downto 0) & a (7 downto 5) when "101",
            a (5 downto 0) & a (7 downto 6) when "110",
            a(6 \text{ downto } 0) \& a(7) \text{ when others}; -- 111
end direct_arch;
```



Better implementation



```
architecture multi_level_arch of rotate_right is
   signal le0_out, le1_out, le2_out:
      std_logic_vector(7 downto 0);
begin
  -- level 0, shift 0 or 1 bit
   le0_out \le a(0) \& a(7 downto 1) when amt(0)='1' else
              a :
  -- level 1, shift 0 or 2 bits
   le1_out <=
     le0_out(1 downto 0) & le0_out(7 downto 2)
        when amt(1)='1' else
     le0_out;
  -- level 2, shift 0 or 4 bits
   le2_out <=
     le1_out(3 downto 0) & le1_out(7 downto 4)
        when amt (2) = 1, else
     le1_out;
   v <= le2_out;
end multi_level_arch;
```

Comparison for n-bit shifter

- Direct implementation
 - n n-to-1 mux
 - vertical strip with O(n²) input wiring
 - Code not so easy to modify
- Staged implementation
 - n*log₂n 2-to-1 mux
 - Rectangular shaped
 - Code easier to modify

5. General examples

- Gray code counter
- Signed addition with status
- Simple combinational multiplier

e.g., Gray code counter

binary code	gray code		
$b_3 b_2 b_1 b_0$	$g_3g_2g_1g_0$		
0000	0000		
0001	0001		
0010	0011		
0011	0010		
0100	0110		
0101	0111		
0110	0101		
0111	0100		
1000	1100		
1001	1101		
1010	1111		
1011	1110		
1100	1010		
1101	1011		
1110	1001		
1111	1000		

gray code	incremented gray code
0000	0001
0001	0011
0011	0010
0010	0110
0110	0111
0111	0101
0101	0100
0100	1100
1100	1101
1101	1111
1111	1110
1110	1010
1010	1011
1011	1001
1001	1000
1000	0000

Direct implementation

```
entity g_inc is
        port (
           g: in std_logic_vector(3 downto 0);
           g1: out std_logic_vector(3 downto 0)
        );
     end g_inc ;
     architecture table_arch of g_inc is
     begin
        with g select
           g1 <= "0001" when "0000",
                 "0011" when "0001",
                 "0010" when "0011",
                 "0110" when "0010",
                 "0111" when "0110".
                 "0101" when "0111",
                 "0100" when "0101",
                 "1100" when "0100",
                 "1101" when "1100",
                 "1111" when "1101",
                 "1110" when "1111",
                 "1010" when "1110",
                 "1011" when "1010",
                 "1001" when "1011",
                 "1000" when "1001",
                 "0000" when others; -- "1000"
    end table_arch;
```

Observation

- Require 2ⁿ rows
- No simple algorithm for gray code increment
- One possible method
 - Gray to binary
 - Increment the binary number
 - Binary to gray

binary code	gray code			
$b_3b_2b_1b_0$	$g_3g_2g_1g_0$			
0000	0000			
0001	0001			
0010	0011			
0011	0010			
0100	0110			
0101	0111			
0110	0101			
0111	0100			
1000	1100			
1001	1101			
1010	1111			
1011	1110			
1100	1010			
1101	1011			
1110	1001			
1111	1000			

binary to gray

$$g_i = b_i \oplus b_{i+1}$$

$$g_3 = b_3 \oplus 0 = b_3$$

$$g_2 = b_2 \oplus b_3$$

$$g_1 = b_1 \oplus b_2$$

$$g_0 = b_0 \oplus b_1$$

gray to binary

$$b_{i} = g_{i} \oplus b_{i+1}$$
 $b_{3} = g_{3} \oplus 0 = g_{3}$
 $b_{2} = g_{2} \oplus b_{3} = g_{2} \oplus g_{3}$
 $b_{1} = g_{1} \oplus b_{2} = g_{1} \oplus g_{2} \oplus g_{3}$
 $b_{0} = g_{0} \oplus b_{1} = g_{0} \oplus g_{1} \oplus g_{2} \oplus g_{3}$

```
architecture compact_arch of g_inc is
  constant WIDTH: integer := 4;
  signal b, b1: std_logic_vector(WIDTH-1 downto 0);
begin
  --- gray to binary
  b <= g xor ('0' & b(WIDTH-1 downto 1));
  --- binary increment
  b1 <= std_logic_vector((unsigned(b)) + 1);
  --- binary to gray
  g1 <= b1 xor ('0' & b1(WIDTH-1 downto 1));
end compact_arch;</pre>
```

e.g., signed addition with status

- Adder with
 - Carry-in: need an extra bit (LSB)
 - Carry-out: need an extra bit (MSB)
 - Overflow:
 - two operands has the same sign but the sum has a different sign

$$overflow = (s_a \cdot s_b \cdot s'_s) + (s'_a \cdot s'_b \cdot s_s)$$

- Zero
- Sign (of the addition result)

```
library ieee;
use ieee.std_logic_1164.all;
use ieee.numeric_std.all;
entity adder_status is
   port (
      a,b: in std_logic_vector(7 downto 0);
      cin: in std_logic;
      sum: out std_logic_vector(7 downto 0);
      cout, zero, overflow, sign: out std_logic
      );
end adder_status;
architecture arch of adder_status is
   signal a_ext, b_ext, sum_ext: signed(9 downto 0);
   signal ovf: std_logic;
   alias sign_a: std_logic is a_ext(8);
   alias sign_b: std_logic is b_ext(8);
   alias sign_s: std_logic is sum_ext(8);
begin
   a_ext <= signed('0' & a & '1');
   b_ext <= signed('0' & b & cin);
   sum_ext <= a_ext + b_ext;
   ovf <= (sign_a and sign_b and (not sign_s)) or
          ((not sign_a) and (not sign_b) and sign_s);
   cout <= sum_ext(9);
   zero <= '1' when (sum_ext(8 downto 1)=0 and ovf='0') else
           '0':
   overflow <= ovf;
   sum <= std_logic_vector(sum_ext(8 downto 1));</pre>
end arch;
```

e.g., simple combinational multiplier

×					a_3 b_3	a_2 b_2	$a_1 \\ b_1$	a_0 b_0	multiplicand multiplier
+		a_3b_3	$a_3b_2 \\ a_2b_3$	a_3b_1 a_2b_2 a_1b_3	$a_2b_1 \\ a_1b_2$	$a_2b_0 \\ a_1b_1 \\ a_0b_2$		a_0b_0	
	y_7	y_6	y_5	y_4	y_3	y_2	y_1	y_0	product

```
library ieee;
use ieee.std_logic_1164.all;
use ieee.numeric_std.all;
entity mult8 is
   port (
      a, b: in std_logic_vector(7 downto 0);
      y: out std_logic_vector(15 downto 0)
   );
end mult8;
architecture comb1_arch of mult8 is
   constant WIDTH: integer:=8;
   signal au, bv0, bv1, bv2, bv3, bv4, bv5, bv6, bv7:
      unsigned(WIDTH-1 downto 0);
   signal p0,p1,p2,p3,p4,p5,p6,p7,prod:
      unsigned(2*WIDTH-1 downto 0);
```

```
begin
   au <= unsigned(a);</pre>
   bv0 \le (others = > b(0));
   bv1 <= (others => b(1));
   bv2 <= (others => b(2));
   bv3 <= (others = > b(3));
   bv4 \le (others = > b(4));
   bv5 \le (others = > b(5));
   bv6 \le (others = > b(6));
   bv7 \le (others = > b(7));
   p0 <= "00000000" & (bv0 and au);
   p1 <= "0000000" & (bv1 and au) & "0";
   p2 <= "000000" & (bv2 and au) & "00";
   p3 <= "00000" & (bv3 and au) & "000";
   p4 <= "0000" & (bv4 and au) & "0000";
   p5 <= "000" & (bv5 and au) & "00000";
   p6 <= "00" & (bv6 and au) & "000000";
   p7 <= "0" & (bv7 and au) & "0000000";
   prod \le ((p0+p1)+(p2+p3))+((p4+p5)+(p6+p7));
   y <= std_logic_vector(prod);</pre>
end comb1_arch;
```