

Question 1

$$A = a_3 a_2 a_1 a_0$$

$$B = b_3 b_2 b_1 b_0$$

Decimal	Binary Sum <small>K 2³ 2² 2¹ 2⁰</small>	BCD Sum <small>C 8 4 2 1</small>
0	0 0000	0 0000
1	0 0001	0 0001
2	0 0010	0 0010
3	0 0011	0 0011
4	0 0100	0 0100
5	0 0101	0 0101
6	0 0110	0 0110
7	0 0111	0 0111
8	0 1000	0 1000
9	0 1001	0 1001
10	0 1010	1 0000
11	0 1011	1 0001
12	0 1100	1 0010
13	0 1101	1 0011
14	0 1110	1 0100
15	0 1111	1 0101
16	1 0000	1 0110
17	1 0001	1 0111
18	1 0010	1 1000
19	1 0011	1 1001

⇒ Max BCD 4 digits

⇒ Max sum

(9+9=18)

⇒ Add A and B with a 4-bit binary adder

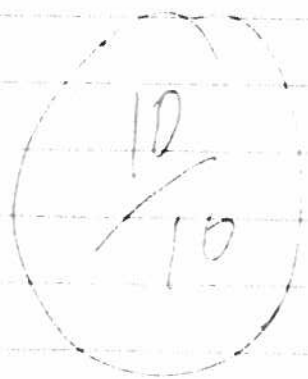
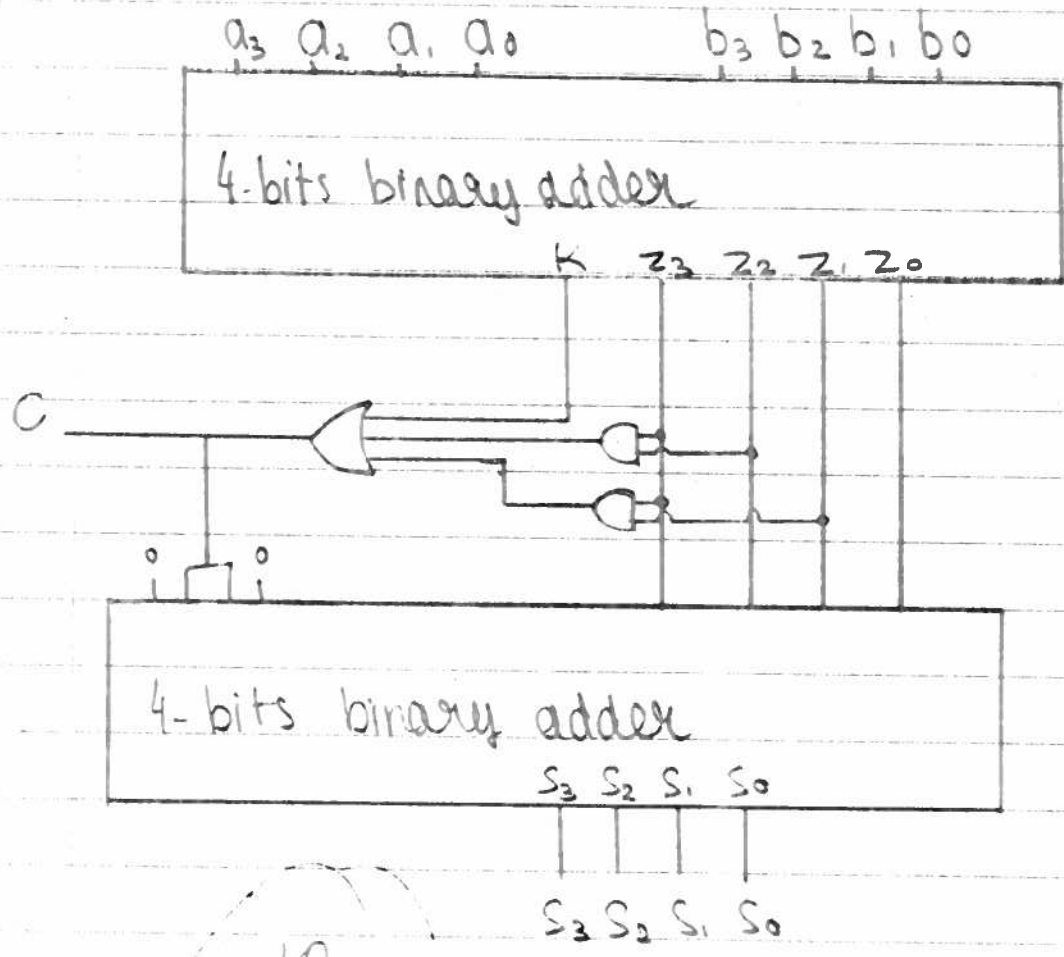
If the sum > 9, add 6

⇒ When C=1, Add 6 to the binary sum
6 = 0110

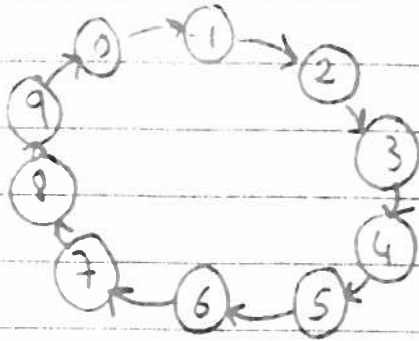
	00	01	11	10
00				
01				
11	1	1	1	1
10			1	1

$$C = K + Z_3 Z_2 + Z_3 Z_1$$

BCD adder



↳ no carry needed in the 2nd adder

Question 2.

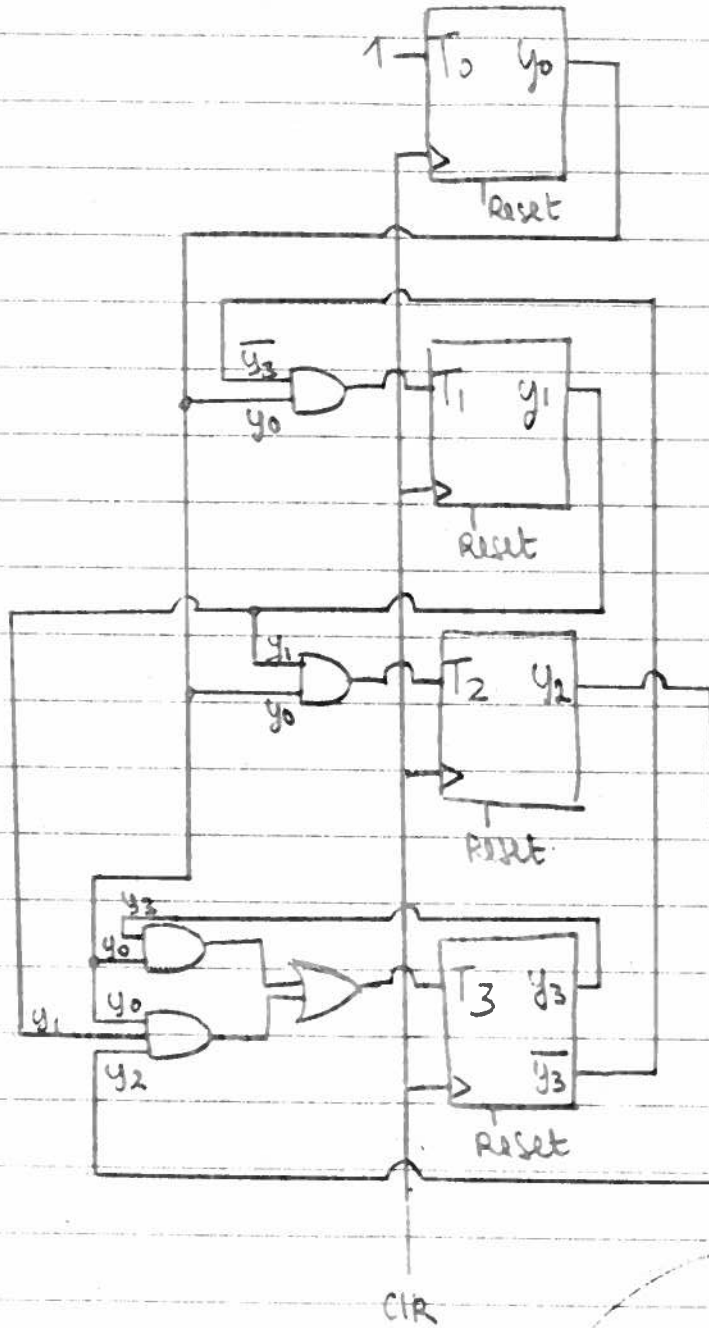
Present State	Next state			
	y_3	y_2	y_1	y_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
0	0	0	0	0

Next: don't care

T Flip Flop:

0 → 0	0
0 → 1	1
1 → 0	1
1 → 1	0

BCD counter



10/10

y_3^+

	y_1	y_0	y_1	y_0	
y_3	y_2	00	01	11	10
00					
01			1		
11	X	X	X	X	
10		1	X	X	

$$T_3 = y_3 y_0 + y_2 y_1 y_0$$

y_2^+

	y_1	y_0	y_1	y_0	
y_3	y_2	00	01	11	10
00			1		
01			1		
11	X	X	X	X	
10			X	X	

$$T_2 = y_1 y_0$$

y_1^+

	y_1	y_0	y_1	y_0	
y_3	y_2	00	01	11	10
00		1	1		
01		1	1		
11	X	X	X	X	
10			X	X	

$$T_1 = \overline{y_3} y_0$$

y_0^+

	y_1	y_0	y_1	y_0	
y_3	y_2	00	01	11	10
00		1	1	1	1
01		1	1	1	1
11	X	X	X	X	
10		1	1	X	X

$$T_0 = 1$$

Question 3

a) $N = N_2 N_1 N_0$ $M = M_2 M_1 M_0$

K	N ₂	N ₁	N ₀		M ₂	M ₁	M ₀
0	0	0	0	0	0	0	1
0	0	0	1	1	0	1	0
0	0	1	0	2	0	1	1
0	0	1	1	3	1	0	0
0	1	0	0	4	1	0	1
0	1	0	1	5	1	1	0
0	1	1	0	6	1	1	1
0	1	1	1	7	X	X	X
1	0	0	0	8	0	1	0
1	0	0	1	9	0	1	1
1	0	1	0	10	1	0	0
1	0	1	1	11	1	0	1
1	1	0	0	12	1	1	0
1	1	0	1	13	1	1	1
1	1	1	0	14	X	X	X
1	1	1	1	15	X	X	X

M₂ K N₂ N₁ N₀

	00	01	11	10
00			1	
01	1	1	X	1
11	1	1	X	X
10			1	1

$M_2 = N_2 + N_1 N_2 + K N_0$
 $= N_2 + N_0 (K + N_1)$

M₁ K N₂ N₁ N₀

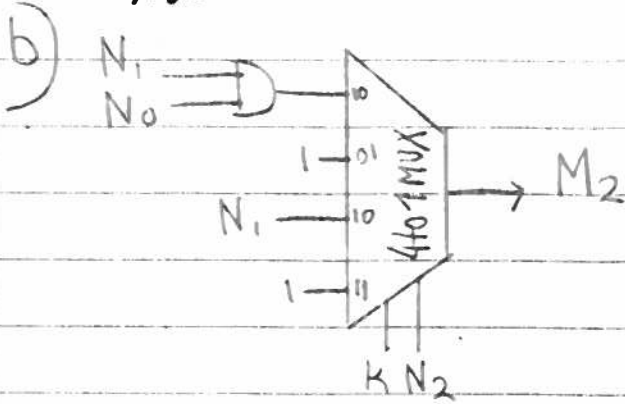
	00	01	11	10
00		1		1
01		1	X	1
11	1	1	X	X
10	1	1		

$M_1 = \overline{N_1} (K + N_0) + K N_1 \overline{N_0}$

M₀ K N₂ N₁ N₀

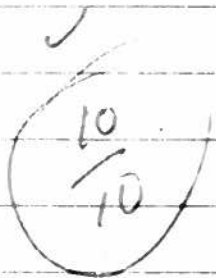
	00	01	11	10
00				1
01	1			1
11		1	X	X
10		1	1	

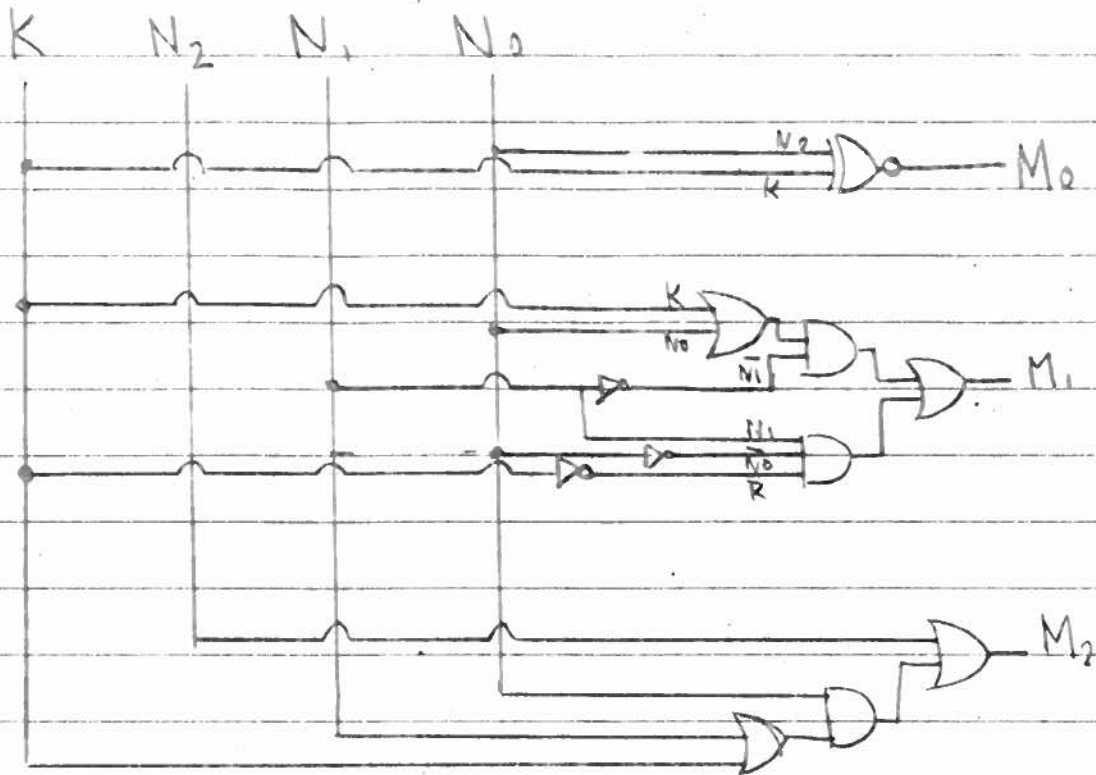
$M_0 = \overline{K N_0} + K N_0$
 $= K \odot N_0$



↓

KN_2	$N_1 N_0$	00	01	11	10
00			1	1	
01			1	1	
11		1	X	X	1
10		1	X	X	1





Question 4

$$\begin{aligned}
 a) \quad f_1(a, b, c) &= \overline{a'c + b'c' + ab} \\
 &= \overline{(a+c')(b+c)(a'+b')} \\
 &= \overline{(ab+ac+bc'+a'b')} \\
 &= \overline{a'ba + a'ac + a'bc' + abb' + ab'c + b'bc'} \\
 &= \overline{abc' + ab'c} \\
 f_2(a, b, c) &= \overline{a'b' + bc + ac'} \\
 &= \overline{(a'b')(bc)(ac')}
 \end{aligned}$$

$$\begin{aligned}
 &= \overline{(a+b)(b+c')(a'+c)} \\
 &= \overline{(ab'+ac+bc')(a'+c)} \\
 &= \overline{aba' + ab'c + ac'a' + ac'c + bc'a' + bc'c} \\
 &= \overline{ab'c + a'bc'} \\
 &= \overline{(ab'c)(a'bc')} = f_1(a, b, c)
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(a, b, c, d) &= \overline{ac' + ab'd + a'b'c + a'cd' + b'c'd'} \\
 &= \overline{ac'(b+b')(d+d') + ab'd(c+c') + a'b'c(d+d') + a'cd'(b+b')} \\
 &= \overline{abc'd + abc'd + abc'd' + abc'd' + ab'cd + ab'cd + ab'cd + ab'cd' + a'bcd + a'bcd + a'bcd + a'bcd'} \\
 &= \overline{a'bcd + a'bcd + a'bcd + a'bcd'}
 \end{aligned}$$

$\frac{1}{2}$

c)

	00	01	11	10
00	0 ₀₀	0 ₀₁	0 ₁₁	0 ₁₀
01	0 ₀₁	1 ₀₁	1 ₁₁	0 ₁₀
11	0 ₁₁	1 ₀₁	1 ₁₁	0 ₁₀
10	0 ₁₀	0 ₀₁	0 ₁₁	0 ₁₀

$$f = \prod M(0, 1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 14)$$

3/3

d) $A = (TE \times TI \times H)'$
 \rightarrow if H means 'it is not a holiday'

if H means 'it is a holiday'

then $A = (TE \cdot TI \cdot \bar{H})'$

2/2

Question 5

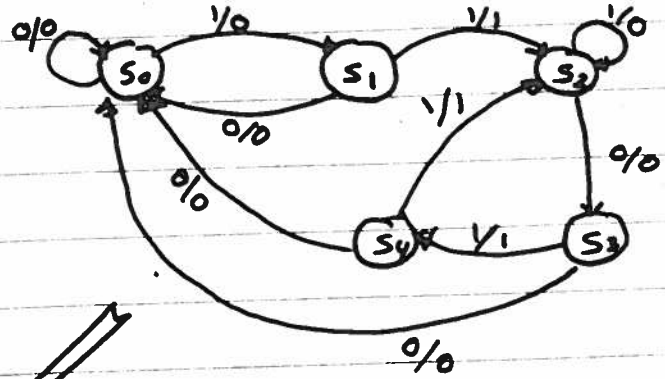
Sequence detector
'1101' and '011' assuming no Read state

$S_0 = 0$

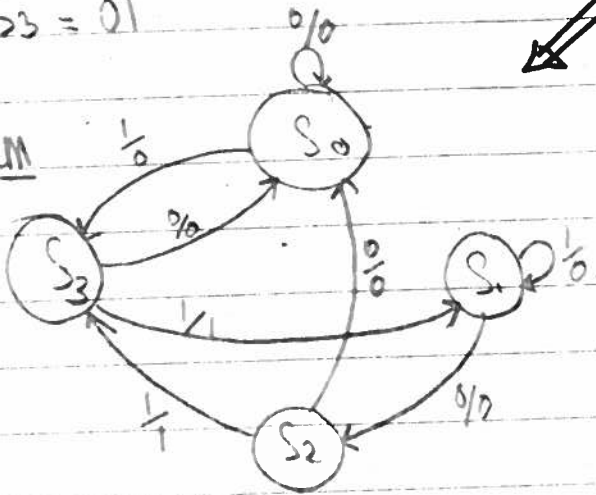
$S_1 = 11$

$S_2 = 110$

$S_3 = 01$



State Diagram



S_0	S_0	0	S_1	0
$\rightarrow S_1$	S_0	0	S_2	1
S_2	S_3	0	S_2	0
S_3	S_0	0	S_4	1
$\rightarrow S_4$	S_0	0	S_2	1

S_1 & S_4 are equal &
 S_4 is replaced by S_1

Transition table

Present state	Next state		Output z	
	$x=0$	$x=1$	$x=0$	$x=1$
S	y_1, y_0	$x=0, y_1^+, y_0^+$ $x=1, y_1^+, y_0^+$		
S_0	0 0	S_0 00 S_3 1 1	0	0
S_1	0 1	S_2 10 S_1 0 1	0	0
S_2	1 0	S_0 00 S_3 1 1	0	1
S_3	1 1	S_0 00 S_1 0 1	0	1

$y_0^+ = x$

$y_1^+ = \bar{x} \bar{y}_1 y_0 + x y_0$

$z = x y_1$

Question 6:

$D = y_1^+$

$D_1 = y_2 \bar{A}$

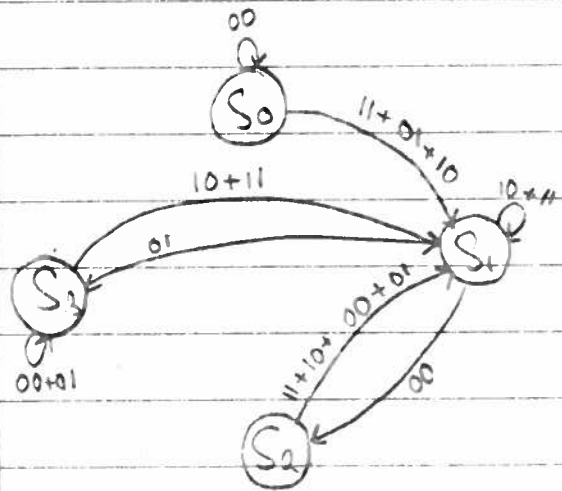
$D_2 = A + B + y_1$

$y_1^+ = y_2 \bar{A}$

$y_2^+ = A + B + y_1$

Present State				Next State	
A	B	y_1	y_2	y_1^+	y_2^+
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	0	1
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	0	1	0	1
1	1	1	0	0	1
1	1	1	1	0	1

State Diagram



✓